

# CHEAT SHEET

- $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$
- $(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$
- $\sum_{k=0}^{\infty} \binom{r+k-1}{r-1} x^k = (1-x)^{-r}$  for  $|x| < 1$
- $\sum_{k=1}^n k = \frac{n(n+1)}{2}$
- $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$
- $\binom{n}{m} + \binom{n}{m+1} = \binom{n+1}{m+1}$
- $\sum_{k=0}^n \binom{n}{k} = 2^n$
- For any integer  $n > 0$  and real  $|r| < 1$ 
  - $\circ \quad \sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$
  - $\circ \quad \sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r}$
  - $\circ \quad \sum_{i=1}^{\infty} r^i = \frac{r}{1-r}$
  - $\circ \quad \sum_{i=1}^n r^i = \frac{r(1-r^n)}{1-r}$

$$\begin{array}{ll} \circ \quad \sum_{i=0}^{\infty} r^i = \frac{1}{1-r} & \circ \quad \sum_{i=1}^{\infty} r^i = \frac{r}{1-r} \\ \circ \quad \sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r} & \circ \quad \sum_{i=1}^n r^i = \frac{r(1-r^n)}{1-r} \end{array}$$

Distribution	$p(y)$	$E(Y)$	$\text{Var}(Y)$	$M(s)$
discrete uniform	$p(y) = \frac{1}{n}$ $y = 1, 2, \dots, n$	$\frac{n+1}{2}$	$\frac{n^2-1}{12}$	$\sum_{x=1}^n \frac{e^{sx}}{n}$
binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y}$ $y = 0, 1, 2, \dots, n$	$np$	$np(1-p)$	$[pe^s + (1-p)]^n$
geometric	$p(y) = p(1-p)^{y-1}$ $y = 1, 2, \dots$	$\frac{1}{p}$	$\frac{(1-p)}{p^2}$	$\frac{pe^s}{1-(1-p)e^s}$
hypergeometric	$p(y) = \frac{\binom{r}{y} \binom{N-r}{n-y}}{\binom{N}{n}}$ $y = 0, 1, \dots, n \text{ if } n \leq r,$ $y = 0, 1, \dots, r \text{ if } n > r$	$\frac{nr}{N}$	$n \left(\frac{r}{N}\right) \left(\frac{N-r}{N}\right) \left(\frac{N-n}{N-1}\right)$	
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!}$ $y = 0, 1, 2, \dots$	$\lambda$	$\lambda$	$\exp[\lambda(e^s - 1)]$
negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$ $y = r, r+1, \dots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{pe^s}{1-(1-p)e^s}\right]^r$

Distribution	$f(y)$	$E(Y)$	$\text{Var}(Y)$	$M(s)$
uniform	$f(y) = \frac{1}{\theta_2 - \theta_1}$ $\theta_1 \leq y \leq \theta_2$	$\frac{\theta_1 + \theta_2}{2}$	$\frac{(\theta_2 - \theta_1)^2}{12}$	$\frac{e^{s\theta_2} - e^{s\theta_1}}{s(\theta_2 - \theta_1)}$
normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2\right]$ $-\infty < y < \infty$	$\mu$	$\sigma^2$	$\exp\left(\mu s + \frac{s^2\sigma^2}{2}\right)$
exponential	$f(y) = \frac{1}{\beta} e^{-y/\beta}$ $\beta > 0$ $0 < y < \infty$	$\beta$	$\beta^2$	$(1 - \beta s)^{-1}$
gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta}$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta s)^{-\alpha}$
chi-square	$f(y) = \frac{y^{(\nu/2)-1} e^{-y/2}}{2^{\nu/2} \Gamma(\nu/2)}$ $y > 0$	$\nu$	$2\nu$	$(1 - 2s)^{-\nu/2}$
beta	$f(y) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1} (1-y)^{\beta-1}$ $0 < y < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	